Bimaximal fermion mixing from the quark and leptonic mixing matrices

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In this paper, we show how the mixing angles of the standard parameterization add when multiplying the quark and leptonic mixing matrices, *i.e.*, we derive explicit sum rules for the quark and leptonic mixing angles. In this connection, we also discuss other recently proposed sum rules for the mixing angles assuming bimaximal fermion mixing. In addition, we find that the present experimental and phenomenological data of the mixing angles naturally fulfill our sum rules, and thus, give rise to bilarge or bimaximal fermion mixing.

I. INTRODUCTION

One of the most fundamental questions in particle physics that still remains to be answered is "What is the mixing of quarks and leptons?". Here this question will be addressed in a phenomenological way. The mixings of both quarks and leptons are described by unitary mixing matrices (relating the fields in the flavor and mass bases). Assuming three generations of quarks and leptons, the quark mixing matrix is known as the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1, 2], whereas the leptonic mixing matrix is mostly known as the Maki–Nakagawa–Sakata (MNS) matrix [3]. Using the so-called standard parameterization of these two matrices [4], it turns out that the CKM matrix has three small mixing angles ($\theta_{12}^{\rm CKM} = 13.0^\circ \pm 0.1^\circ$, $\theta_{13}^{\rm CKM} = 0.2^\circ \pm 0.1^\circ$, and $\theta_{23}^{\rm CKM} = 2.4^\circ \pm 0.1^\circ$ [5]) and the MNS matrix has two large mixing angles as well as probably one small mixing angle ($\theta_{12}^{\rm MNS} = 33.2^{\circ +4.9^\circ}_{\ -4.6^\circ}$, $\theta_{13}^{\rm MNS} = 0 \pm 12.5^\circ$, and $\theta_{23}^{\rm MNS} = 45.0^{\circ +10.6^\circ}_{\ -9.4^\circ}$ [6]). In addition to the mixing angles, the standard parameterization also contains a CP-violating phase $\theta_{\rm CP}$ [29]. In the quark sector, CP violation has been measured restricting the CP-violating phase to $\delta_{\rm CP}^{\rm CKM} = 1.05 \pm 0.24$ [5], whereas in the leptonic sector, until now, CP violation has not been measured leading to a completely undetermined value of the corresponding CP-violating phase. Recently, it has been suggested that the mixing angles that parametrize the mixing matrices could fulfill the following relations [7, 8]: $\theta_{12}^{\rm CKM} + \theta_{12}^{\rm MNS} = \frac{\pi}{4}$, $\theta_{13}^{\rm CKM} \sim \theta_{13}^{\rm MNS} = \mathcal{O}(\lambda^3)$, and $\theta_{23}^{\rm CKM} + \theta_{23}^{\rm MNS} = \frac{\pi}{4}$, where λ is the Wolfenstein parameter [9]. It should be noted that the first relation, which relates the Cabibbo angle and the solar mixing angle, was proposed a long time ago [10]. Nevertheless, this relation has recently been discussed in the literature [7, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19] to a great extent and it is now referred to as the quark-lepton complementarity (QLC) relation. Next, these three above relations suggest that the bilarge leptonic mixing and the small quark mixing could be related to some exact fundamental bimaximal mixing. Thus, a flavor symmetry is needed in order to describe such a mixing. It should be pointed out that the values of the mixing parameters of course depend on the parameterization used. A relation for the mixing parameters in a specific representation of a mixing matrix is not necessarily the same in another representation.

In this paper, we argue that bimaximal mixing naturally appears when multiplying the quark and leptonic mixing matrices coming from recent experimental and phenomenological data.

The paper is organized as follows: In Sec. II, we derive explicit sum rules for the quark and leptonic mixing angles resulting in total mixing angles for a general fermion mixing. Next, we show that the present experimental and phenomenological values of the quark and leptonic mixing angles naturally lead to bilarge or maybe even bimaximal fermion mixing. Then, in Sec. III, we discuss the earlier obtained results on QLC as well as bilarge or bimaximal fermion mixing. Finally, in Sec. IV, we summarize our results and present our conclusions.

II. EXPLICIT SUM RULES FOR QUARK AND LEPTONIC MIXING ANGLES

The standard parameterization of a 3×3 unitary mixing matrix is given by [4]

$$U = {\rm e}^{{\rm i}\lambda_7 \theta_{23}} U_\delta {\rm e}^{{\rm i}\lambda_5 \theta_{13}} U_\delta^\dagger {\rm e}^{{\rm i}\lambda_2 \theta_{12}} = O_{23} U_\delta O_{13} U_\delta^\dagger O_{12}$$

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$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}c_{23} \end{pmatrix}, \tag{1}$$

where λ_i $(i = 1, 2, \ldots, 8)$ are the Gell-Mann matrices, O_{ij} is the orthogonal rotation matrix in the ij-plane which depends on the mixing angle θ_{ij}, $U_\delta = \mathrm{diag}(1, 1, e^{i\delta_{\mathrm{CP}}})$, δ_{CP} being the Dirac CP-violating phase, $s_{ij} \equiv \sin\theta_{ij}$, and $c_{ij} \equiv \cos\theta_{ij}$. This parameterization is used for both the quark and leptonic sectors. Now, we denote the Cabibbo–Kobayashi–Maskawa quark mixing matrix by V_{CKM}, whereas we denote the Maki–Nakagawa–Sakata leptonic mixing matrix by U_{MNS}. In general, in the three-flavor case, there are also two Majorana CP-violating phases, which can be introduced in a mixing matrix U by replacing the matrix with $U\Phi$, where $\Phi = \mathrm{diag}(1, e^{i\phi_1}, e^{i\phi_2})$ with ϕ_1 and ϕ_2 being the Majorana CP-violating phases. However, these two phases do not affect neutrino oscillations and will therefore not be considered here.

The present experimental and phenomenological values of the moduli of the quark and leptonic mixing matrices are given by [5, 6]

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.9739 \div 0.9751 & 0.221 \div 0.227 & 0.0029 \div 0.0045 \\ 0.221 \div 0.227 & 0.9730 \div 0.9744 & 0.039 \div 0.044 \\ 0.0048 \div 0.014 & 0.037 \div 0.043 & 0.9990 \div 0.9992 \end{pmatrix}, \tag{2}$$

$$|U_{\mathrm{MNS}}| = \begin{pmatrix} 0.79 \div 0.88 & 0.48 \div 0.62 & < 0.22 \\ 0.14 \div 0.64 & 0.36 \div 0.82 & 0.58 \div 0.84 \\ 0.14 \div 0.64 & 0.36 \div 0.82 & 0.58 \div 0.84 \end{pmatrix}, \tag{3}$$

where the values of the modulus of the quark mixing matrix are the 90 % confidence level ranges, whereas the values of the modulus of the leptonic mixing matrix are the 3σ (99.7 % confidence level) ranges. In fact, the modulus of the leptonic mixing matrix has been constructed from the following quantities $0.23 \le \sin^2\theta_{12}^{\mathrm{MNS}} \le 0.38$, $\sin^2\theta_{13}^{\mathrm{MNS}} \le 0.047$, and $0.34 \le \sin^2\theta_{23}^{\mathrm{MNS}} \le 0.68$ [6]. Another recent phenomenological value of this matrix is given by [20]

$$|U_{\mathrm{MNS}}| = \begin{pmatrix} 0.79 \div 0.88 & 0.47 \div 0.61 & < 0.20 \\ 0.19 \div 0.52 & 0.42 \div 0.73 & 0.58 \div 0.82 \\ 0.20 \div 0.53 & 0.44 \div 0.74 & 0.56 \div 0.81 \end{pmatrix}. \tag{4}$$

Note that the ranges of the quark mixing matrix have been determined using eight constraints from tree-level processes, which means that there will be no information on the CP-violating phase in the quark sector, and thus, the values of this phase can be set to zero, *i.e.*, $\delta_{\mathrm{CP}}^{\mathrm{CKM}} = 0$. Actually, in order to obtain information on the CP-violating phase in this sector, we need to take into account additional loop-level processes as additional constraints [5]. In addition, note that there is no knowledge about the value of the CP-violating phase in the leptonic sector, *i.e.*, the value of $\delta_{\mathrm{CP}}^{\mathrm{MNS}}$ is allowed to lie in the whole interval $[0, 2\pi)$. From the two matrices V_{CKM} and U_{MNS}, assuming that the CP-violating phases in both the quark and leptonic sectors are equal to zero, *i.e.*, $\delta_{\mathrm{CP}}^{\mathrm{CKM}} = 0$ and $\delta_{\mathrm{CP}}^{\mathrm{MNS}} = 0$, as well as using the above ranges of the matrix elements in Eqs. (2) and (3), we can read off the mixing angles to be [21]

$$\begin{cases} \theta_{12}^{\mathrm{CKM}} = 13.0° \pm 0.1°, \\ \theta_{13}^{\mathrm{CKM}} = 0.2° \pm 0.1°, \\ \theta_{23}^{\mathrm{CKM}} = 2.4° \pm 0.1°, \end{cases} \quad \begin{cases} \theta_{12}^{\mathrm{MNS}} = 33.2° \pm 4.9°, \\ \theta_{13}^{\mathrm{MNS}} = 0 \pm 12.5°, \\ \theta_{23}^{\mathrm{MNS}} = 45.0° \pm 10.6°. \end{cases}$$

Note that the matrix elements in Eq. (4) yield the following values for the leptonic mixing angles: $\theta_{12}^{\mathrm{MNS}} = 32.9° \pm 4.8°$, $\theta_{13}^{\mathrm{MNS}} = 0 \pm 11.5°$, and $\theta_{23}^{\mathrm{MNS}} = 45.6° \pm 10.1°$, which are more or less the same as the ones obtained above.

In order to investigate mixing on a more fundamental level, we will add the quark and leptonic mixings. This is performed by multiplying the corresponding mixing matrices, which is motivated by quark-lepton unification. However, there are two possibilities of multiplying these matrices either $U_{\mathrm{MNS}}V_{\mathrm{CKM}}$ or $V_{\mathrm{CKM}}U_{\mathrm{MNS}}$, which have been investigated in Ref. [8]. Note that these two resulting unitary mixing matrices do not commute, which means that the two possible ways of multiplying the matrices will give different results. Furthermore, note that the mixing angles do not simply add in the trivial way as in the case of 2×2 unitary (or orthogonal) mixing matrices, *i.e.*, $\theta = \theta^{\mathrm{MNS}} + \theta^{\mathrm{CKM}} = \theta^{\mathrm{CKM}} + \theta^{\mathrm{MNS}}$.

Multiplying the two unitary mixing matrices in the following order

$$W_1 = U_{\mathrm{MNS}}V_{\mathrm{CKM}} \tag{5}$$

and assuming that the quark mixing angles are small compared with the leptonic mixing angles, we obtain series

expansions for the total mixing angles of the W_1 matrix

$$\theta_{12} \simeq \theta_{12}^{\mathrm{MNS}} + \theta_{12}^{\mathrm{CKM}} + \left(s_{12}^{\mathrm{MNS}}\theta_{13}^{\mathrm{CKM}} - c_{12}^{\mathrm{MNS}}\theta_{23}^{\mathrm{CKM}}\right)\tan\theta_{13}^{\mathrm{MNS}}, \tag{6}$$

$$\theta_{13} \simeq \theta_{13}^{\mathrm{MNS}} + c_{12}^{\mathrm{MNS}}\theta_{13}^{\mathrm{CKM}} + s_{12}^{\mathrm{MNS}}\theta_{23}^{\mathrm{CKM}}, \tag{7}$$

$$\theta_{23} \simeq \theta_{23}^{\mathrm{MNS}} + \left(c_{12}^{\mathrm{MNS}}\theta_{23}^{\mathrm{CKM}} - s_{12}^{\mathrm{MNS}}\theta_{13}^{\mathrm{CKM}}\right)\sec\theta_{13}^{\mathrm{MNS}}, \tag{8}$$

which are sum rules valid upto first order in the small quark mixing angles. On the other hand, performing the multiplication in the opposite order, *i.e.*,

$$W_2 = V_{\mathrm{CKM}}U_{\mathrm{MNS}}, \tag{9}$$

we find for the total mixing angles of the W_2 matrix

$$\theta_{12} \simeq \theta_{12}^{\mathrm{MNS}} + \left(c_{23}^{\mathrm{MNS}}\theta_{12}^{\mathrm{CKM}} - s_{23}^{\mathrm{MNS}}\theta_{13}^{\mathrm{CKM}}\right)\sec\theta_{13}^{\mathrm{MNS}}, \tag{10}$$

$$\theta_{13} \simeq \theta_{13}^{\mathrm{MNS}} + s_{23}^{\mathrm{MNS}}\theta_{12}^{\mathrm{CKM}} + c_{23}^{\mathrm{MNS}}\theta_{13}^{\mathrm{CKM}}, \tag{11}$$

$$\theta_{23} \simeq \theta_{23}^{\mathrm{MNS}} + \theta_{23}^{\mathrm{CKM}} + \left(s_{23}^{\mathrm{MNS}}\theta_{13}^{\mathrm{CKM}} - c_{23}^{\mathrm{MNS}}\theta_{12}^{\mathrm{CKM}}\right)\tan\theta_{13}^{\mathrm{MNS}}. \tag{12}$$

Multiplying the two mixing matrices in the order $V_{\mathrm{CKM}}U_{\mathrm{MNS}}$ has also been discussed in Refs. [16, 22]. Note that the mixing angles for the W_2 matrix in Eqs. (10)-(12) can be obtained from the mixing angles for the W_1 matrix in Eqs. (6)-(8) by replacing the "12" indices with the "23" indices, and vice versa. Therefore, in the first case, the total mixing angle θ_{12} is linearly dependent on all quark mixing angles and the other total mixing angles θ_{13} and θ_{23} are only linearly dependent on $\theta_{13}^{\mathrm{CKM}}$ and $\theta_{23}^{\mathrm{CKM}}$, whereas in the second case, the total mixing angle θ_{23} is linearly dependent on all quark mixing angles and the other total mixing angles θ_{12} and θ_{13} are only linearly dependent on $\theta_{12}^{\mathrm{CKM}}$ and $\theta_{13}^{\mathrm{CKM}}$. Furthermore, if the CP-violating phases are assumed to be non-zero, then the resulting formulas for the total mixing angles will be much more complicated expressions.

In general, without any specific parameterization of the mixing matrices, but instead using the matrix elements of the mixing matrices, Eqs. (5) and (9) can be written as $\sum_{k=1}^{3}(U_{\mathrm{MNS}})_{ik}(V_{\mathrm{CKM}})_{kj} = (W_1)_{ij}$ and $\sum_{k=1}^{3}(V_{\mathrm{CKM}})_{ik}(U_{\mathrm{MNS}})_{kj} = (W_2)_{ij}$, respectively, where i and j are fixed ($i, j = 1, 2, 3$). Assuming that the CKM mixing matrix is close to the 3×3 identity matrix, *i.e.*, $(V_{\mathrm{CKM}})_{ij} = \delta_{ij} + \epsilon_{ij}$, where δ_{ij} is Kronecker's delta and ϵ_{ij}'s are small, which should correspond to the quark mixing angles being small, we obtain the following relations $(U_{\mathrm{MNS}})_{ij} + \sum_{k=1}^{3}(U_{\mathrm{MNS}})_{ik}\epsilon_{kj} = (W_1)_{ij}$ and $(U_{\mathrm{MNS}})_{ij} + \sum_{k=1}^{3}(U_{\mathrm{MNS}})_{kj}\epsilon_{ik} = (W_2)_{ij}$. From these relations, inserting a specific parameterization of the mixing matrices, it is then possible to derive similar sum rules to those obtained in Eqs. (6)-(8) and (10)-(12) for this specific parameterization.

Inserting the best-fit values including the ranges of the quark and leptonic mixing angles into the formulas of the total mixing angles Eqs. (6)-(8), we obtain the following values for the mixing angles

$$\theta_{12} = 46.2° \pm 5.4°, \quad \theta_{13} = 1.5° \pm 12.8°, \quad \text{and} \quad \theta_{23} = 46.9° \pm 10.9°,$$

which should be compared with the "exact" numerical values that actually are exactly the same. This means that the expansion formulas are very accurate. Observe that the error propagation is completely dominated by the errors in the leptonic mixing angles and that the contribution from the errors in the quark mixing angles is, in principle, negligible. Similar, inserting the best-fit values including the ranges into Eqs. (10)-(12), we find that

$$\theta_{12} = 42.3° \pm 6.8°, \quad \theta_{13} = 9.3° \pm 14.3°, \quad \text{and} \quad \theta_{23} = 47.4° \pm 12.7°,$$

which also should be compared with the "exact" numerical values that are $\theta_{12} = 42.3° \pm 8.7°$, $\theta_{13} = 9.3° \pm 14.2°$, and $\theta_{23} = 46.7° \pm 12.4°$. Again, the agreement between the results of the expansion formulas and the "exact" numerical calculations is very good. However, the errors are slightly larger than in the previous case, but again completely dominated by the contribution from the errors in the leptonic mixing angles.

It is interesting to note that in both cases a bilarge (*i.e.*, two mixing angles are close to maximal or maximal and one angle is small or zero) mixing pattern arises, which means that $\theta_{12} \simeq 45°$, $\theta_{13} \simeq 0$, and $\theta_{23} \simeq 45°$. In addition, both cases are even consistent with a bimaximal mixing pattern, where $\theta_{12} = 45°$, $\theta_{13} \simeq 0$, and $\theta_{23} = 45°$.

Finally, there is actually a third possible way (maybe even more natural) to combine the two mixing matrices, which would be to take a linear combination of the two matrices W_1 and W_2, *i.e.*, $W_3 = aW_1 + bW_2$, where a and b are constants. However, this would lead to a total mixing matrix, which would not be unitary. Therefore, this possibility will not be discussed any further.

III.   DISCUSSION OF EARLIER RESULTS

The QLC relation indicates that there could be a quark-lepton symmetry or even quark-lepton unification based on the Pati–Salam model [23, 24] such as SU(5) [or SU(5) and SO(10)] GUT. Recently, this relation has been generally investigated by Minakata and Smirnov [11]. If not (numerically) accidental, then a solid motivation for the QLC relation needs to be found and it has to be rigorously experimentally tested. Furthermore, renormalization group equations for running of the QLC relation have been derived and analyzed in Refs. [22, 25]. The result of the analysis suggest that if the QLC relation is assumed at high energies, then it does not necessarily mean that the QLC relation is fulfilled at low energies. Note that the leptonic mixing runs faster than the quark mixing due to the fact that the leptonic mixing angles are larger than the quark mixing angles.

In addition, Raidal [7] has suggested three relations $\theta_{12}^{\mathrm{CKM}}+\theta_{12}^{\mathrm{MNS}}=\frac{\pi}{4}$, $\theta_{13}^{\mathrm{CKM}}\sim\theta_{13}^{\mathrm{MNS}}=\mathcal{O}(\lambda^3)$, and $\theta_{23}^{\mathrm{CKM}}+\theta_{23}^{\mathrm{MNS}}=\frac{\pi}{4}$ motivated by a flavor symmetry, which indicate that there could exist a simple relation between the quark and leptonic mixings. In principle, the relations proposed by Raidal serve as a generalization of the QLC relation. In the derivation of these relations, it has been assumed that the quark mixing matrix describes the deviation of the leptonic mixing matrix from exactly bimaximal, which he concludes should be due to some unknown underlying non-Abelian flavor physics. Using the three relations, Li and Ma [8] have performed several test of these relations. Especially, they have parameterized the MNS mixing matrix with an assumed bimaximal mixing matrix as well as the Wolfenstein parameters [9] of the CKM mixing matrix. Using this parameterization, they have calculated both possible products of the two mixing matrices and found theoretically that the relation $U_{\mathrm{MNS}}V_{\mathrm{CKM}}=W_{\mathrm{bimaximal}}$ is in better agreement with Raidal's relations than the relation $V_{\mathrm{CKM}}U_{\mathrm{MNS}}=W_{\mathrm{bimaximal}}$, where $W_{\mathrm{bimaximal}}$ is the assumed bimaximal mixing matrix. Note that the second relation $V_{\mathrm{CKM}}U_{\mathrm{MNS}}=W_{\mathrm{bimaximal}}$ has also been discussed in Refs. [14, 26, 27, 28].

However, in this paper, we have not assumed the exact relations of the quark and leptonic mixing matrices used by Li and Ma, but we have instead phenomenologically investigated the matrix products $U_{\mathrm{MNS}}V_{\mathrm{CKM}}$ and $V_{\mathrm{CKM}}U_{\mathrm{MNS}}$, and hence, we have derived series expansion formulas for the total mixing angles upto first order in the small quark mixing angles. Using the present allowed experimental and phenomenological ranges of the quark and leptonic mixing angles, it has been found that bimaximal (or at least bilarge) mixing naturally appears, *i.e.*, we have not assumed that the product of the mixing matrices is a bimaximal mixing matrix.

IV.   SUMMARY AND CONCLUSIONS

In summary, we have derived series expansion formulas for the fermionic mixing angles in terms of the quark and leptonic mixing angles. These formulas are explicit sum rules for the quark and leptonic mixing angles in a unified fermionic picture of quark and leptonic mixing. The formulas are valid upto first order in the small quark mixing angles. However, due to the smallness of the quark mixing angles, the formulas are indeed very accurate. In addition, we have shown that it turns out, using these sum rules, that present data naturally lead to bilarge or bimaximal fermion mixing. It is important to note in this paper that we have not assumed bilarge or bimaximal fermionic mixing, but bilarge or bimaximal fermionic mixing is purely a result of combining the mixings stemming from experimental and phenomenological data of quarks and leptons. The way to test the results presented in this paper will be to use data from future precision measurements of the quark and leptonic mixing angles such as data from B physics and neutrino oscillation experiments.

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[29] Note that a general 3×3 unitary mixing matrix can also contain two Majorana CP-violating phases. However, these two phases are not measurable using neutrino oscillations and, in general, both Dirac and Majorana CP-violating phases will make the situation much more complicated than without them.